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The Hierarchical Organization of Cognitive Abilities: Restoring General Intelligence Through the Use of Linear Structural Relations (LISREL)

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While considerable support has been obtained for the distinction, due to Cattell and Horn, between the broad factors fluid intelligence (Gf) and crystallized intelligence (Gc), there is also some evidence that Gf is equivalent to the factor of general intelligence (*g*). This hypothesis is investigated by using LISREL to specify higher-order models in reanalyses of three sets of psychometric data from subjects 11, 13, and 15 years old, respectively. The three studies unanimously showed Gf to be equivalent with a general factor. The discussion is centered upon the use of LISREL and the differences in results between the exploratory and confirmatory approaches to factor analysis.

Introduction

Many empirical studies now support second-order simple structure factor analytic distinctions consistent with the Cattell-Horn theory of fluid and crystallized intelligence, the Gf-Gc theory (Crawford & Nirmal, 1976; Cattell, 1963, 1967a, 1967b; Cattell & Horn, 1978; Horn & Bramble, 1976; Horn & Cattell, 1966; Rossman & Horn, 1972; Shucard & Horn, 1972; Undheim, 1976, 1978, 1981a). However, hierarchical order analyses of such data using a Schmid-Leiman hierarchical ortogonalization approach (Schmid & Leiman, 1957) have suggested that fluid intelligence (Gf) as a second-order factor may be identical with the general factor (*g*) (Undheim, 1981b). In a study of 15-year olds where fluid intelligence was represented by a broad selection of reasoning tests (figural, symbolic and semantic content) in accordance with the Cattell-Horn theory, the Schmid-Leiman transformation resulted in a *g*-factor with Gf tasks having

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loadings approaching unity so that the Gf factor disappeared in the orthogonalized solution, while group factors of crystallized intelligence (Gc), visualization (Gv), and speed (Gs) remained. On the basis of applying such transformation to three other sets of data, Undheim suggested (1981b) that the empirical equivalence of the Gf-factor with the *g*-factor in a matrix of cognitive tests may be dependent on a quite broad sampling of primary factors.

Recently, however, Gustafsson (1984) has argued that the method of linear structural relations, LISREL, developed by Jöreskog and Sörbom (1978, 1981) may be particularly suited for testing hierarchical models of intelligence. The LISREL procedure was applied to performances of about 1200 Swedish 12-year olds on 16 tests (Gustafsson, 1984). The measures selected defined fairly broad second-order factors of Gf, Gc, and Gv. A third-order analysis using LISREL showed that the Gf factor coincided with the *g*-factor. The results were thus consistent with the findings of Undheim (1981b), although the latter study also included a measure of broad speediness (Gs).

One purpose of this paper is to further investigate the robustness and generality of the above findings of hierarchical organization using the LISREL approach. Three sets of data were selected for reanalysis which were originally analyzed by simple structure factor analysis in the course of studying second-order factor concepts (Undheim, 1976, 1978, 1981a) and subsequently formed the base for Undheim's attempt to "restore" general intelligence as a central and viable concept (1981b, c, d) through the Schmid-Leiman orthogonalization procedure (Undheim 1981b).

Another purpose is to study the LISREL method itself, and particularly how to use and evaluate the results of this method in analyzing hierarchical order relations among latent variables. Method studies of simple structure factor analysis over the past 50 years have resulted in a number of rules-of-the-thumb for such analysis. There is undoubtedly a need for similar knowledge regarding adequate use of LISREL.

General Methodology

In a higher-order analysis the estimates of the relations among the lower-order factors provide the basis for identification of the higher-order factors, so it is of course essential that the intercorrelations among the lower-order factors are objectively and correctly estimated. One major advantage of the LISREL method in the estimation and testing of hierarchical models is that the estimates of relations between latent variables are unique, while in exploratory analysis the degree of obliqueness of the solution is influenced by which particular method of rotation is used and by which parameter values are chosen to govern the process of rotation. Another important advantage of the LISREL method is that since the model is constrained by the hypothesized relations among variables, fewer indicators are needed to identify a factor. Thus, while in exploratory factor analysis at least three or four tests are needed to identify each factor, in LISREL two tests frequently suffice.

These advantages of the LISREL method, along with those that follow from the hypothesis testing capabilities of the system, should make it a suitable technique for investigating hierarchical models of ability. Use of LISREL is not without its problems, however. Unless variables are carefully selected a large and heavily constrained LISREL model rarely fits the data, if it converges at all. This makes it necessary to modify the model, which most frequently is done on the basis of the relations that are present in the sample. Unfortunately, however, such modifications disturb the inferential characteristics of the method, and may invalidate the use of LISREL for testing statistical hypotheses (Cliff, 1983).

The rather large test-batteries reanalyzed here have not been assembled with the purpose to enable fitting of LISREL models, so it may be expected that many modifications are needed to achieve even a nominal fit, with all the ensuing problems. This will cause introduction of relations which may not prove replicable, it does disturb the nominal levels of significance, and it might be argued that it may bias the models in favor of the main hypothesis that Gf equals *g*. The data have, therefore, in parallel been subjected to two kinds of analyses: one (approach A) in which models have freely been modified to optimize fit; and one (approach B) where test-specific influences are minimized before the LISREL analyses through simple summation of test scores before the analysis and where ad hoc modifications are used restrictively.

The LISREL analysis according to approach A was performed in the way described by Gustafsson (1984). The procedure started with a model involving first-order factors only and ended with a model with factors at three levels. In the initial step at each level a theoretically derived model was fitted. If the fit of this model was poor, as evaluated with the chi-square goodness-of-fit test, it was modified to achieve an acceptable fit before proceeding to the next higher level. Modifications were indicated by the modification indices provided by LISREL, but the changes made were in each case also influenced by findings previously established using multiple factor analysis. In these modifications, models were frequently fitted for subsets of the variables and the sub-models were then pieced together into one model (see Gustafsson, 1984). Since this process is quite elaborate it will be impossible to document each of the steps in the present context.

In approach B the first-order analysis was "eliminated" by summing standard scores on tests to represent primary factors. Here the LISREL analysis thus starts at a higher level in the hierarchy of factors. The idea was to sidestep modifications due to test-specific influences, in order to enable testing the hierarchical model with a minimum of modifications. This shortcut is, of course, at the expense of being able to "weed out" the specifics of tests in defining primary factors as latent variables, which is not, however, the primary purpose here. In the analyses to be described the summation of variables was accomplished by calculating new correlation coefficients that would reflect the correlational patterns of unweighted sums of composites (see Guilford & Hoepfner, 1971, p. 45–46). The result is empirically equivalent to summation of standard scores.

Study I: 11-Year-Olds

This study is a reanalysis of data presented in Undheim (1976). The subjects were 144 fourth-grade children attending three primary schools in Trondheim, Norway. The 68 girls and 76 boys had a mean age of 10 years and 10 months. The test variables consisted of 12 subtests from a preliminary Norwegian version of the WISC (Wechsler Intelligence Scale for Children) as well as 23 group tests modeled after Thurstone (1938) and Guilford and Hoepfner (1971). In the original analysis (Undheim, 1976), a total of 24 variables (6 WISC subtests and 18 group tests) were included in the exploratory factor analysis to establish primary factors; another 6 tests were included in the second-order analysis to help define broad factors of Gf, Gc, Gs, and Gv.

The LISREL analysis is based on the set of 24 tests, supplemented with two other variables—the WISC subtests Information and Comprehension—so that the Gc domain should have a somewhat broader variable base. The variables, along with their primary and secondary factor involvement as established by previous research, are presented

Table 1

Tests used in the study of 11-year olds (Study I), and the factor belongingness of the tests according to previous research

Test	Primary Factor	Broad Factor
1. WISC - Block Design (BD)	Visualization (Vz)	Gv
2. WISC - Object Assembly (OA)	"	"
3. Block Counting (BC)	"	"
4. Paper Form Board (PFB)	"	"
5. Punched Holes (PH)	"	"
6. Card Rotations (CR)	Spatial Relations (S)	"
7. Flag Rotations (FR)	"	"
8. Street Gestalt Part I (SG1)	Speed of Closure (Cs)	"
9. Street Gestalt Part II (SG2)	"	"
10. Figure Analogies (FA)	Figural Relations (CFR)	Gf
11. Figure Classification (FC)	"	"
12. Figure Exclusion (FE)	"	"
13. Figure Matrix (FM)	"	"
14. WISC - Digit Span Forw (DSF)	Memory Span (Ms)	"
15. WISC - Digit Span Backw (DSB)	"	"
16. Arithmetic Reasoning (AR)	General Reasoning (R)	Gf and/or Gc
17. Necessary Facts (NF)	"	"
18. WISC - Arithmetic Reas (WAR)	"	"
19. WISC - Vocabulary (VOC)	Verbal Comprehension (V)	Gc
20. Synonyms (SYN)	"	"
21. WISC - Information (INF)	General Information (Inf)	"
22. WISC - Comprehension (COM)	"	"
23. Letter Identification (LI)	Speed of Symbol Discr (ESU)	Gs
24. Symbol Identities (SI)	"	"
25. Number Addition (NA)	Number (N)	"
26. Number Multiplication (NM)	"	"

Note. The Figural Relations factor comprises both CFR and CFC in Guilford's terminology.

in Table 1. Additional details on the sample of subjects and test variables are presented in Undheim (1976). In Table 2 the correlation matrix for these 26 tests is presented.

Results: Analysis A

Since all primary factors were represented by two or more tests, a full hierarchical model with factors at the primary, secondary, and tertiary level was formulated, following the step-wise procedure described in the section on General Methodology. The final model, which is presented in Figure 1, had, according to the overall likelihood-ratio test, an acceptable fit to the data (chi-square = 285.77, df = 270, $p <$ .24). In the Figure the tests are shown enclosed in squares while the factors are enclosed in circles. Straight arrows indicate direction of influence from higher to lower level variables, and the estimates of these parameters may be interpreted as standardized factor loadings. The curved bidirectional arrows indicate correlation among variables

Table 2

Correlations between the tests in Study I

	BD	OA	BC	PFB	PH	CR	FR	SG1	SG2	FA	FC	FE	FM	DSF	DSB	AR
1. BD	100															
2. OA	65	100														
3. BC	47	36	100													
4. PFB	41	39	33	100												
5. PH	57	49	45	26	100											
6. CR	49	41	45	46	38	100										
7. FR	42	35	39	31	42	50	100									
8. SG1	33	33	23	27	35	21	25	100								
9. SG2	29	32	18	19	35	19	25	73	100							
10. FA	41	35	29	25	35	42	32	27	20	100						
11. FC	33	30	22	38	30	32	12	25	21	35	100					
12. FE	42	51	34	33	40	42	27	31	28	50	37	100				
13. FM	51	48	42	34	51	37	30	37	26	53	35	55	100			
14. DSF	17	21	09	09	21	11	11	12	05	19	09	20	19	100		
15. DSB	05	25	09	10	10	17	01	18	20	22	21	32	22	30	100	
16. AR	49	34	40	32	39	40	29	20	10	51	34	47	51	12	23	100
17. NF	50	40	31	28	37	42	37	25	18	41	33	47	38	20	15	69
18. WAR	36	28	26	17	28	21	30	21	10	40	23	37	36	20	18	66
19. VOC	44	44	36	31	34	31	28	41	37	47	47	45	47	14	24	64
20. SYN	30	27	17	19	19	20	24	18	13	33	37	27	22	19	17	54
21. INF	36	36	31	17	29	22	18	26	23	33	43	35	28	00	17	54
22. COM	39	33	28	21	29	17	13	29	29	23	40	28	31	19	21	42
23. LI	19	18	15	13	24	24	16	14	18	23	22	28	20	04	35	15
24. SI	16	15	21	09	12	25	15	05	06	26	32	37	25	13	27	24
25. NA	12	10	22	30	24	24	23	06	00	21	32	24	31	15	19	38
26. NM	25	10	19	18	21	33	25	11	04	26	28	22	21	14	18	40

	NF	WAR	VOC	SYN	INF	COM	LI	SI	NA	NM
17. NF	100									
18. WAR	57	100								
19. VOC	64	57	100							
20. SYN	52	45	69	100						
21. INF	57	52	74	53	100					
22. COM	41	43	58	42	65	100				
23. LI	20	14	34	22	18	03	100			
24. SI	22	25	29	34	19	04	53	100		
25. NA	37	35	23	26	20	13	29	52	100	
26. NM	32	36	26	30	21	10	34	50	51	100

Note. Decimal points omitted.

at the same level.

The model contains 10 primary factors and 4 second-order factors with relations that closely correspond to previous research findings:

1. Below Gv there are three primary factors: Vz, S, and Cs. There is also a relationship between Gv and CFR, which certainly is due to the figural content of the CFR-tests.

2. Below Gf there are factors representing CFR, R, and Ms.

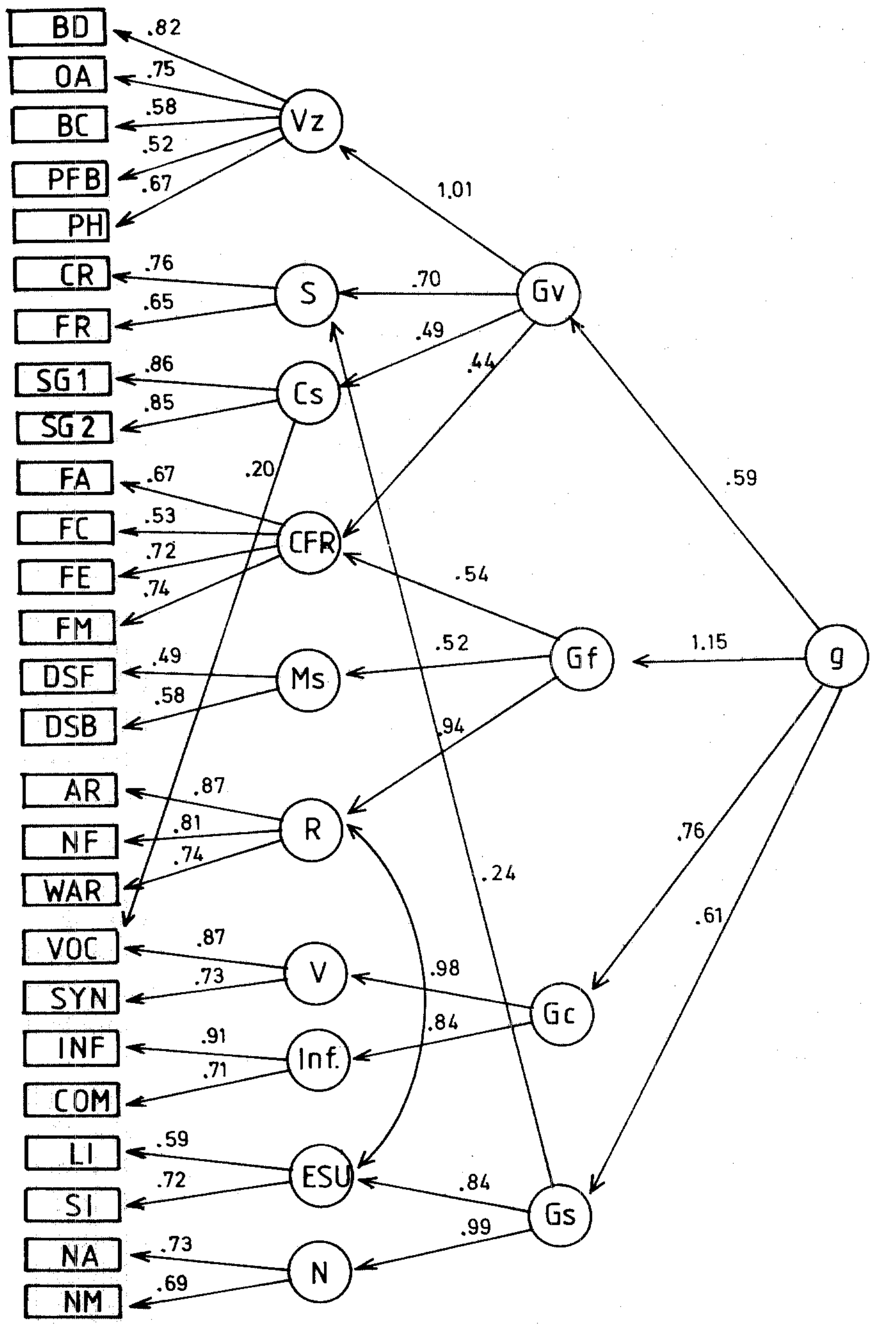

Figure 1. Final LISREL model for 11-year olds, Analysis A (Study I).

3. Below Gc there are two factors, one of which is interpreted to

represent V and the other a factor called General Information (Inf), affecting performance on the two WISC subtests Information and Comprehension.

4. In the domain of Gs there are primary factors representing N and Speed of Symbol Discrimination/Identification (ESU), and there is also a relation between Gs and the S-factor, which accounts for performance on speeded spatial tests.

In order to achieve a statistically non-significant fit it also proved necessary to allow for covariances between the specific parts of tests in nine cases (CR, PFB; LI, VOC; NA, PFB; SI, PH; FM, SYN; FM, INF; DSF, INF; DSB, BD; DSF, STG2). These effects are likely to represent transient factors, such as test-ordering at administration, and minor primary factors. No attempt has therefore been made to interpret them, nor are they represented in Figure 1.

The hypothesis of major interest is that Gf is equivalent to the *g*-factor, which is the case when there is a standardized loading of unity of Gf in *g*. As may be seen in Figure 1 the loading of Gf in *g* is in fact larger than unity (1.15), so the model represents a Heywood case. The *t*-value for the negative residual variance in Gf is not significant, however, ($t = -1.66$), so the aberrant loading may be interpreted as a chance effect. Since all the other broad factors have loadings significantly lower than unity in the *g*-factor, these results may be interpreted as supporting the hypothesis of equivalence between Gf and *g*.

Results: Analysis B

In this analysis unweighted combinations of test variables were used as indicators of 10 primary factors according to the scheme of Table 1. The LISREL model thus was set up with the 4 broad factors Gf, Gc, Gv, and Gs as first-level latent variables and with one latent variable as a second-order factor.

The chi-square value of 60.62 (df = 30, $p < 0.001$) indicates that the fit of this model is not very good. Nonetheless, it is noteworthy that the correlation between *g* and Gf is 1.08 and significantly higher than that obtained with any other of the 3 broad factors. Imposing the constraint of equality between *g* and Gf did not significantly worsen the fit (chi-square = 1.01, df = 1, $p < 0.315$). Furthermore, by introducing a correlated error between indicators of primary factors N and R, as suggested by the modification indices of LISREL, the fit becomes acceptable at the 5% level (chi-square = 48.81, df = 29, $p < 0.058$), while Gf still correlates about 1.0 with *g* (actually 1.01). This

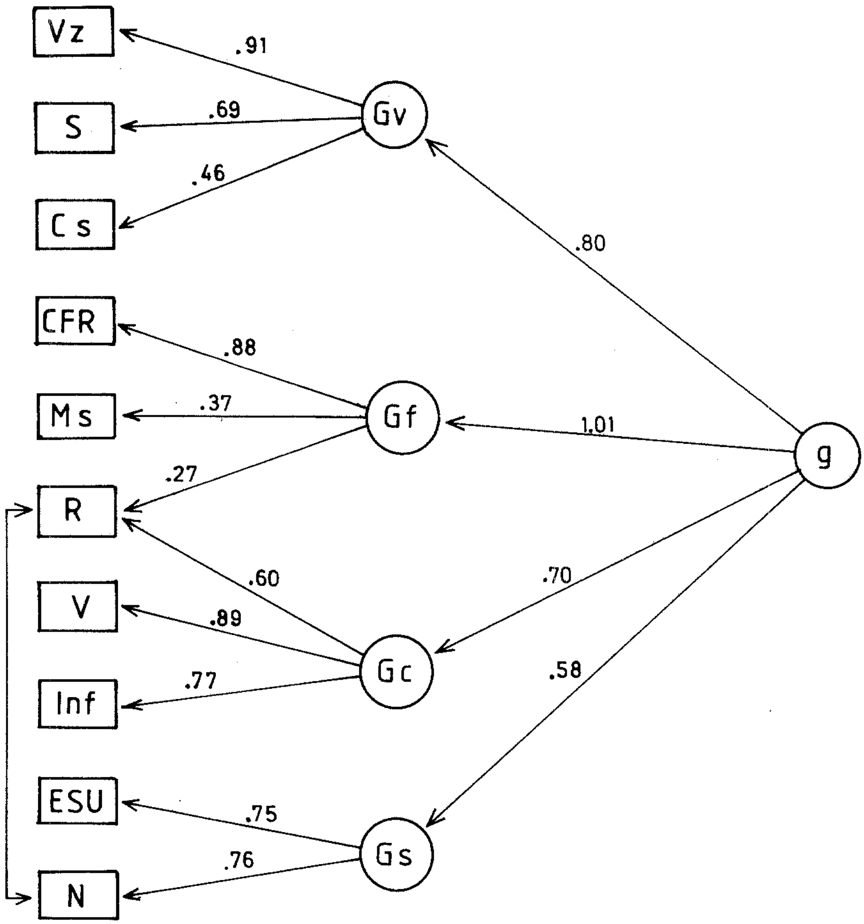

Figure 2. Final LISREL model for 11-year olds, Analysis B (Study I).

modification certainly is very reasonable on the basis of previous factor analytic work. The final solution is presented in Figure 2.

Study II: 13-Year-Olds

This study is a reanalysis of data presented in Undheim (1978). The subjects were 149 sixth-grade children attending two public schools in Trondheim, Norway. The 92 girls and 57 boys had a mean age of 12 years 10 months. Thirty tests were administered, selected to represent primary factors that would allow identification of the broad ability factors Gf, Gc, Gv, Gs, and Gr. There were one or two tests for

Table 3

Tests used in the study of 13-year olds (Study II), and the factor belongingness of the tests according to previous research

Test	Primary Factor	Broad Factor
1. Word Fluency (WF)	Word Fluency (Fw)	Gr
2. Word Listing (WL)	"	"
3. Anagram Fluency (AF)	"	"
4. Ideational Fluency (IF)	Ideational fluency (Fi)	"
5. Uses (USE)	"	"
6. Consequences (CON)	"	"
7. Synonyms (SYN)	Verbal Comprehension (V)	Gc
8. Antonyms (ANT)	"	"
9. Verbal Classification (VC)	Semantic Classific (CMC)	"
10. Sentence Selection (SS)	Formal Reasoning (Rs)	Gf and/or Gc
11. Necessary Facts (NF)	General Reasoning (R)	"
12. Arithmetic Reasoning (AR)	"	"
13. Circle Reasoning (CIR)	Induction (I)	Gf
14. Number Series (NS)	"	"
15. Matrices (MAT)	Figural Relations (CFR)	"
16. Series (SER)	"	"
17. Figure Analysis (FA)	"	"
18. Card Rotation (CR)	Spatial Relations (S)	Gv
19. Block Counting (BC)	Visualization (Vz)	"
20. Paper Form Board (PFB)	"	"
21. Punched Holes (PH)	"	"
22. Surface Development (SD)	"	"
23. Street Gestalt Complet (SG)	Speed of Closure (Cs)	"
24. Mutilated Words (MW)	"	"
25. Letter Identification (LI)	Speed of Symbol Discr (ESU)	Gs
26. Symbol Identities (SI)	"	"
27. Identical Forms (IFO)	Perceptual Speed (P)	"
28. Marking Speed (MS)	Motor Speed	"
29. Addition (ADD)	Number (N)	"
30. Multiplication (MUL)	"	"

15 primary factors previously recognized by French, Ekstrom, and Price (1963), Guilford (1967), Horn (1966) or Pawlik (1966).

Additional details on the sample of subjects and test variables are presented in Undheim (1978), along with the results of principal factor analysis and rotations according to several simple structure procedures. The findings of Undheim (1978) regarding broad ability factors confirmed previous results concerning the loading patterns of Gf, Gc, Gv, Gs, and Gr.

Table 3 shows the tests, along with their hypothesized primary factor relations, and Table 4 presents the correlation matrix among the 30 tests.

Results: Analysis A

The analysis proceeded in the same fashion as in Study I, starting with simpler models deduced from the information in Table 3, which

Table 4
Correlations between the tests in Study II

	WF	WL	AF	IF	USE	CON	SYN	ANT	VC	SS	NF	AR	CIR	NS	MAT	SER
1. WF	100															
2. WL	65	100														
3. AF	66	56	100													
4. IF	60	53	44	100												
5. USE	53	37	50	44	100											
6. CON	46	36	44	53	58	100										
7. SYN	42	30	40	43	39	48	100									
8. ANT	48	37	46	46	44	53	72	100								
9. VC	52	50	45	50	47	42	51	54	100							
10. SS	49	29	50	43	48	56	62	70	47	100						
11. NF	43	37	41	51	44	54	63	66	46	70	100					
12. AR	43	39	39	44	43	44	51	60	45	67	73	100				
13. CIR	32	31	33	37	35	40	37	50	37	51	60	64	100			
14. NS	51	41	46	44	45	40	44	53	47	59	56	60	42	100		
15. MAT	37	34	36	30	34	29	38	36	34	40	41	52	27	50	100	
16. SER	41	34	37	38	43	35	33	44	42	40	42	47	47	49	45	100
17. FA	36	32	44	38	37	38	41	50	33	47	50	58	41	52	55	53
18. CR	27	11	24	22	31	29	38	36	22	41	44	34	23	41	17	25
19. BC	34	27	22	37	27	36	30	37	29	39	48	47	34	35	31	30
20. PFB	35	35	23	36	26	33	35	37	37	32	50	50	35	41	38	36
21. PH	53	39	40	41	42	34	41	42	41	43	45	52	42	54	40	41
22. SD	35	36	37	37	30	36	37	41	40	42	47	56	38	41	42	43
23. SG	25	18	18	21	23	23	14	16	12	29	23	24	18	17	12	16
24. MW	36	34	25	37	30	29	32	30	32	25	34	32	16	35	26	38
25. LI	24	25	41	13	18	10	33	22	26	20	17	22	07	39	26	16
26. SI	28	37	26	30	27	18	35	30	41	25	29	24	18	31	28	20
27. IFO	37	32	33	34	27	25	32	30	36	26	33	31	29	30	25	24
28. MS	35	37	41	36	34	35	36	31	32	26	26	21	18	31	24	28
29. ADD	50	50	46	51	37	28	38	40	50	38	43	50	33	47	41	38
30. MUL	36	39	38	36	28	21	31	30	37	21	30	28	28	42	43	28

	FA	CR	BC	PFB	PH	SD	SG	MW	LI	SI	IFO	MS	ADD	MUL
17. FA	100													
18. CR	27	100												
19. BC	27	38	100											
20. PFB	31	43	57	100										
21. PH	39	50	57	59	100									
22. SD	40	34	49	48	58	100								
23. SG	10	27	37	28	30	41	100							
24. MW	26	32	43	35	38	39	27	100						
25. LI	21	19	07	21	21	18	-03	14	100					
26. SI	21	30	15	21	23	17	04	21	48	00				
27. IFO	30	42	32	41	43	31	23	32	25	33	100			
28. MS	27	17	15	21	23	21	10	21	29	30	31	100		
29. ADD	41	24	38	37	48	34	14	23	34	56	31	37	100	
30. MUL	35	11	11	28	33	22	03	15	32	45	18	33	58	100

Note. Decimal points omitted.

were then fitted together into one model. The final model is presented in Figure 2. According to the overall chi-square test this model had an acceptable fit (chi-square = 370.92, df = 329, $p < .06$).

As may be seen in Figure 3 the final model included the five hypothesized broad abilities Gf, Gc, Gv, Gr, and Gs, along with the *g*-factor. The model is not a full hierarchical model, however, since some of the broad factors appear as first-order factors.

The major characteristics of this model may be described as follows:

1. Within the domain of broad fluency two primary factors are identified (Fw and Fi), very much in the manner expected from the original classification of tests.

2. Within the Gc-domain only one first-order factor was identifiable. This factor includes the Verbal Comprehension tests, and also the Sentence Selection test, which originally was hypothesized to belong with the Gf domain. This does, however, make the factor somewhat broader in scope, and makes it more appropriate to apply the label Gc for the factor.

3. The list of tests classified with the Gf-domain (see Table 3) is rather heterogeneous and an acceptable fit was obtained only by leaving out of the analysis the tests Circle Reasoning and Number Series, and by organizing the other variables in a Cognition of Figural Relations (CFR) factor and a factor of General Reasoning (R).

4. Within the area of broad speediness (Gs) it proved possible to identify three primary factors: Symbol Discrimination (ESU), Number (N) and Marking Speed. The latter is probably a rather odd and sample-specific constellation; the exploratory factor analysis showed Identical Forms to have most of its variance in Gv, and Marking Speed to show strong affinity to Gr (Undheim, 1978).

5. In the area of broad visualization (Gv) a one-factor model showed acceptable fit, despite the fact that two primaries (Vs and Cs) were represented by two or more tests. One likely explanation for this may be the fact that the sample consisted of a majority of young females, who tend to show less differentiation with the domain of visual abilities.

6. At the second-order level the model identifies the factors Gr, Gs, and Gf in a manner very much like what is to be expected from the hypotheses expressed in Table 3. It may be noted, however, that Gs had weak relations with more primaries than was originally expected.

The estimate of the relationship between Gf and the teritiary *g*-factor was very close to unity (1.01), and below unity for all the other broad abilities. Thus, this study too provides support for the hypothesis that the factor of Gf is identical with the *g*-factor.

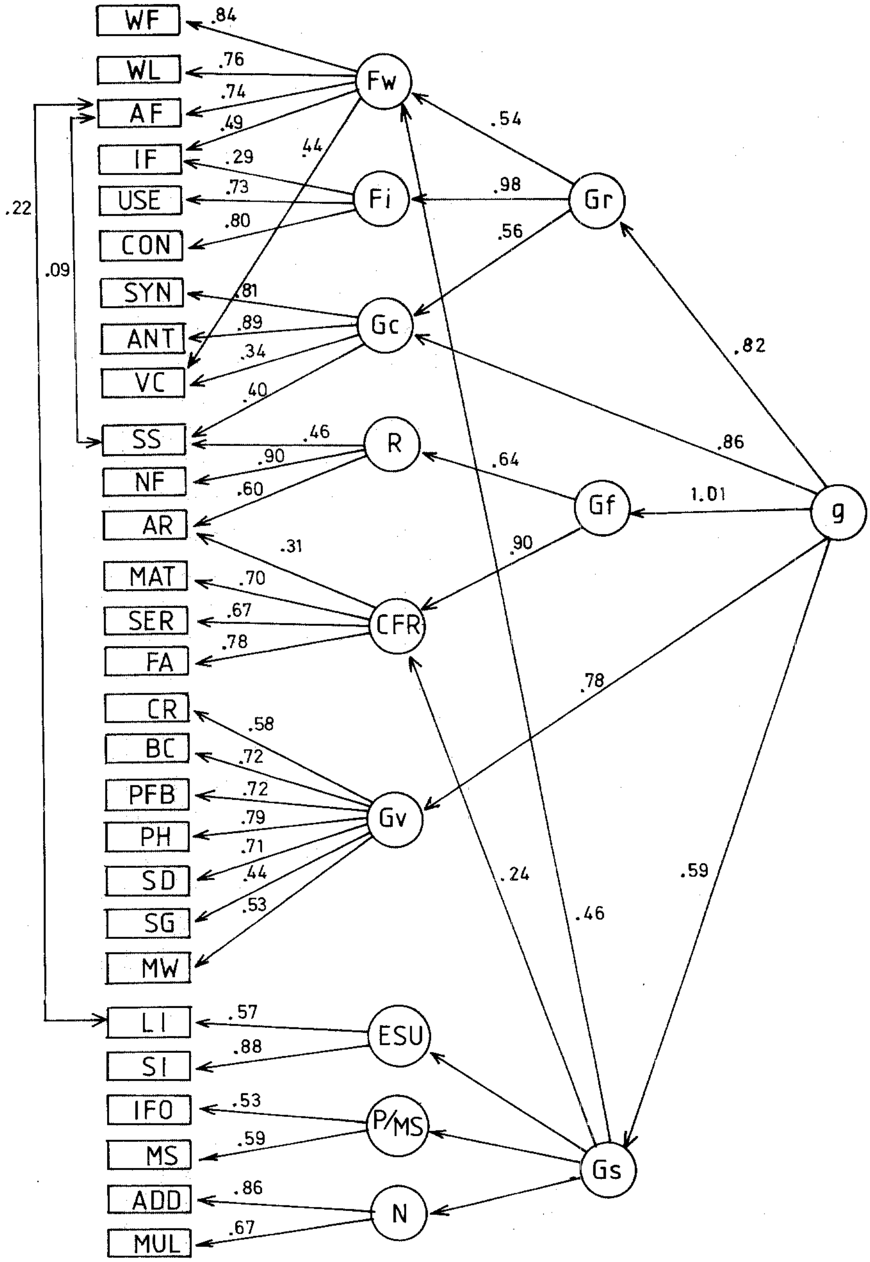

Figure 3. Final LISREL model for 13-year olds, Analysis A (Study II).

Results: Analysis B

In analysis B unweighted combinations of test variables were used to create indicators of 10 primary factors, while 5 other primary factors had only one test indicator. We expected that this might result in a greater need for modifications of the LISREL model since specific influences are not balanced by a summing procedure in the "estimation" of factors. In order to reduce this problem somewhat, 2 variables that were not necessary in the identification of broad factors (Verbal Classification and Marking Speed) were eliminated.

The LISREL model was set up according to the scheme of Table 3 with 5 broad factors Gf, Gc, Gv, Gs, and Gr as first-level latent variables and with one latent variable at the highest level, the *g*-factor. The chi-square value of 112.72 (df = 59, $p < 0.000$) indicates that the model fit is far from acceptable, and even though *g* has its highest correlation with Gf, it is possible to reject the hypothesis that *g* has a perfect relationship with Gf at the 5% level. The modification indices of LISREL pointed at a relation between primary factor P (Identical Forms) and the Gv factor, and a relation between Fw and the Gs factor. Introducing these two modifications resulted in a model that could not be rejected at the 1% level (chi-square = 79.81, df = 57, $p <$ 0.025). In this model, which is presented in Figure 4, the estimate of the correlation between *g* and Gf was .95, which is not significantly different from unity (chi-square = 2.53, df = 1, $p < 0.112$).

Of the modifications introduced, the relationship of P (Identical Forms) to Gv is quite reasonable considering the figural content of the only P-test available. In the previous broad factor study, the test, in fact, had its main loading on Gv (Undheim, 1978). As for the relation of Fw to Gs, speed elements are obviously present in most fluency tests. In the previous broad factor study, this was manifested as a substantial correlation between Gr and Gs in oblique solutions (Undheim, 1978).

Study III: 15-Year-Olds

The study is a reanalysis of data originally presented by Undheim (1981a). The subjects were 148 children from eighth and ninth grade in two public schools of Trondheim, Norway. The 103 girls and 43 boys had a mean age of 15 years. Twenty-one tests were administered; the tests were selected to represent primary factors that would allow for the identification of the broad ability factors Gf, Gc, Gv, Gs, and Gr. More detailed information on the tests and the sample is supplied by Undheim (1981a).

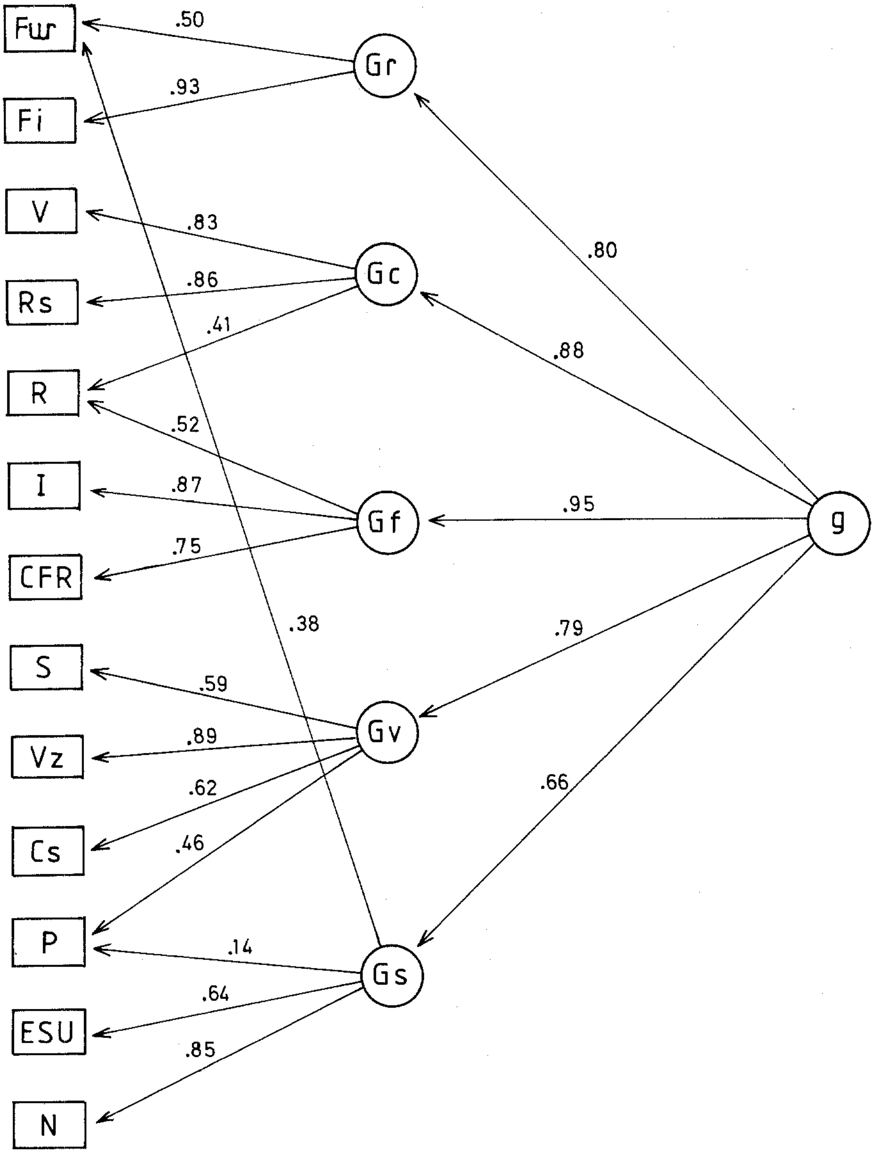

Figure 4. Final LISREL model for 13-year olds, Analysis B (Study II).

The variables, along with their factor belongingness as established by previous research, are presented in Table 5. In Table 6 the correlation matrix for the 21 tests is presented.

From the list of primary factors represented by the tests (see Table 5) it is clear that in most cases each primary is represented by only

Table 5

Tests used in the study of 15-year olds (Study III), and the factor belongingness of the tests according to previous research

Test	Primary Factor	Broad Factor
1. Card Rotation (CR)	Spatial Relations (S)	Gv
2. Surface Development (SD)	Visualization (Vz)	"
3. Street Gestalt Compl (SG)	Speed of Closure (Cs)	"
4. Hidden Figures (HF)	Flexibility of Closure (Cf)	"
5. Coding (COD)	Perceptul Speed (P)	Gs
6. Symbol Identities (SI)	Speed of Symbol Discr (ESU)	"
7. Identical Forms (IFO)	Perceptual Speed (P)	"
8. Number Additions (ADD)	Number (N)	"
9. Circle Reasoning (CIR)	Induction (I)	Gf
10. Letter Series (LS)	"	"
11. Matrices (MAT)	Figural Relations (CFR)	"
12. Verbal Analogies (VA)	Semantic Relations (CMR)	Gf and/or Gc
13. Necessary Facts (NF)	General Reasoning (R)	"
14. Arithmetic Reasoning (AR)	"	"
15. Sentence Selection (SS)	Formal Reasoning (Rs)	"
16. Vocabulary (VOC)	Verbal Comprehension (V)	Gc
17. Information (INF)	General Information (Inf)	"
18. Word Fluency (WF)	Word Fluency (Fw)	Gr
19. Synonym Fluency (SF)	Associational Fluency (Fa)	"
20. Ideational Fluency (IF)	Ideational Fluency (Fi)	"
21. Uses (USE)	"	"

onetest. Since normally at least two tests are required to identify a primary factor, this makes it impossible to specify a full hierarchical model with factors at the primary, secondary, and tertiary levels.

Results: Analysis A

In the original exploratory analysis of these data Undheim (1981a) got support for five broad ability factors in accordance with the hypothesized pattern. This also indicates that it will not be possible to find much of a differentiated structure at the level of primary factors with the present data. Some primary factors, however, are represented by more than one test. This is true for Induction, Ideational Fluency, and General Reasoning, and the rather powerful LISREL technique may be expected to be able to identify these primary factors.

A confirmatory model for these tests may therefore be hypothesized to consist of a mixture of undifferentiated second-order factors and some primary factors. In order, however, to keep the LISREL model as simple as possible the general strategy was to start with a one-factor model for each domain of broad abilities.

According to the overall likelihood-ratio test the final model had a rather good fit to data (chi-square = 134.29, df = 120, $p < .18$). This

Table 6

Correlations between the tests in Study III

		CR	SD	SG	HF	COD	SI	IFO	ADD	CIR	LS	MAT	VA	NF	AR	SS	VOC
1.	CR	100															
2.	SD	40	100														
3.	SG	27	46	100													
4.	HF	41	58	44	100												
5.	COD	27	12	24	43	100											
6.	SI	26	08	19	36	56	100										
7.	IFO	42	32	25	43	48	47	100									
8.	ADD	30	13	08	37	57	57	44	100								
9.	CIR	42	39	24	47	27	27	27	30	100							
10.	LS	51	48	37	60	43	45	50	45	63	100						
11.	MAT	45	40	37	41	26	24	39	33	49	57	100					
12.	VA	33	48	21	46	22	28	31	32	46	56	46	100				
13.	NF	43	45	22	47	22	30	34	24	49	55	36	52	100			
14.	AR	54	49	21	47	13	31	25	31	55	56	42	48	59	100		
15.	SS	38	43	32	48	33	45	37	34	50	63	41	56	59	63	100	
16.	VOC	35	37	25	35	18	32	24	22	38	37	31	43	48	53	61	100
17.	INF	40	41	22	34	11	24	26	14	36	41	36	42	45	64	51	60
18.	WF	25	18	28	39	35	46	29	46	20	37	22	25	27	25	41	48
19.	SF	19	22	24	24	18	31	35	19	24	24	28	27	30	18	43	39
20.	IF	09	29	32	30	07	14	17	09	15	22	19	29	21	16	34	35
21.	USE	17	34	31	27	27	15	31	22	24	24	19	31	29	22	30	34

		INF	WF	SF	IF	USE
17.	INF	100				
18.	WF	29	100			
19.	SF	29	45	100		
20.	IF	52	45	40	100	
21.	USE	29	33	38	38	100

Note. Decimal points omitted.

model is presented in Figure 5.

To arrive at the model presented in Figure 5 several modifications of the original hypotheses had to be made:

1. For Gv the test Card Rotation had to be left out in order to obtain an acceptable fit to a one-factor model. The most likely reason for this is that there are relationships between specific parts of the test (and/or primary factors) and these covariances would require further factors to account for them.

2. For the seven Gf-tests a one-factor model fitted very poorly. Various two-factor models were tested, but only after the Verbal Analogies test was left out was an acceptable two-factor model obtained. In this model there is one factor of Induction—defined by Circle Reasoning, Letter Series, and Matrices—and a composite Reasoning

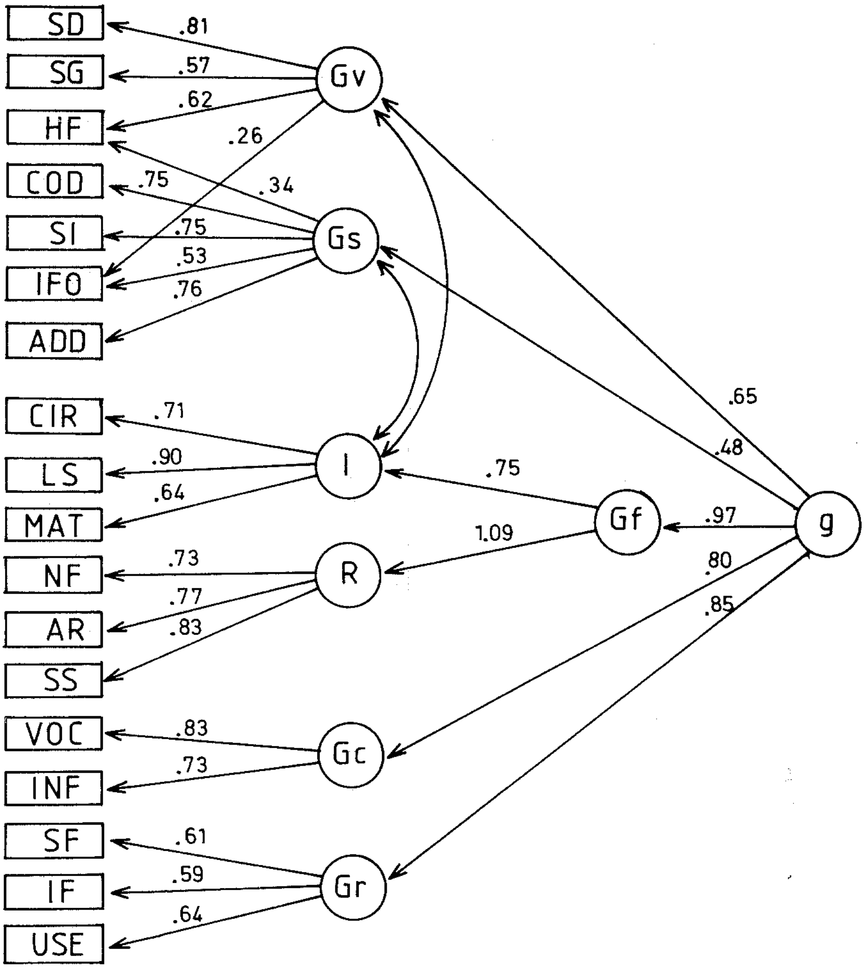

Figure 5. Final LISREL model for 15-year olds, Analysis A (Study III).

factor-defined by Necessary facts, Arithmetic Reasoning, and Sentence Selection. This latter factor may be viewed as comprised of the two primary factors Formal Reasoning and General Reasoning.

3. For Gr just as for Gv it proved impossible to fit a one-factor model for the four tests hypothesized to belong with the domain. However, after the Word Fluency test was left out, the remaining three tests did fit well into a one-factor pattern.

4. Of the broad ability factors hypothesized for the matrix the model thus identifies Gv, Gs, Gc, and Gr as first-order factors, while the Gf-factor technically as well as theoretically appears as a second-order factor. In spite of the fact, however, that most of the broad

abilities are represented as first-order factors they can rather unambiguously be interpreted as higher-order constructs. The factor at the apex of the hierarchy (*g*) is, therefore, assumed to affect the 5 broad factors, whether these are represented as first- or second-order factors.

The estimates of the degree of relationship between *g* and the broad factors again support the hypothesis that Gf is equivalent with the *g*-factor: For Gf the coefficient is .97 while for all the other broad factors it is lower.

Results: Analysis B

In this test battery only 3 primary factors had two or more marker variables which could be summed as indices of these factors. A total of 15 factors had only one marker variable. Thus, the sampling of variables does not permit the balancing of specific influences intended in this alternative LISREL procedure.

Nonetheless, the model as depicted in Table 5 was tested, with 5 second-order factors Gf, Gc, Gv, Gs, and Gr as first-level latent variables and with one latent variable at the highest level, the *g*-factor. The chi-square value of 214.23 (df = 111, $p < .000$) indicates that the model fit is not at all acceptable. In this case *g* had its closest relationship to Gv, followed by Gf. Furthermore, several modifications based on the modification indices, although seemingly reasonable enough, did not result in a level of fit that was acceptable. Thus, this alternative procedure provided no answer to the substantial question under scrutiny.

To effect a reduction of the influence of test-specific factors it is necessary in this case to enter the hierarchy at an even higher level, through obtaining summed indices of the broad factors. One problem is, however, that some of the tests/factors are hypothesized to be influenced by both Gf and Gc. To solve this problem the test Sentence Selection was classified with the Gc-tests, while all other of these tests were classified with Gf.

The LISREL-model in this case is, of course, extremely simple, and just involves fitting a one-factor model to the five observed variables Gf, Gc, Gv, Gs, and Gr. The fit of this model was rather poor, however (chi-square = 27.07, df = 5, $p < .00$). In this model Gf had the strongest relationship with *g* (.89). According to the modification indices a covariance between the specific parts of Gf and Gr was the reason for the poor fit. Freeing this parameter, the overall goodness-of-fit test was non-significant (chi-square = 3.80, df = 4, $p < .47$). The model is presented in Figure 6.

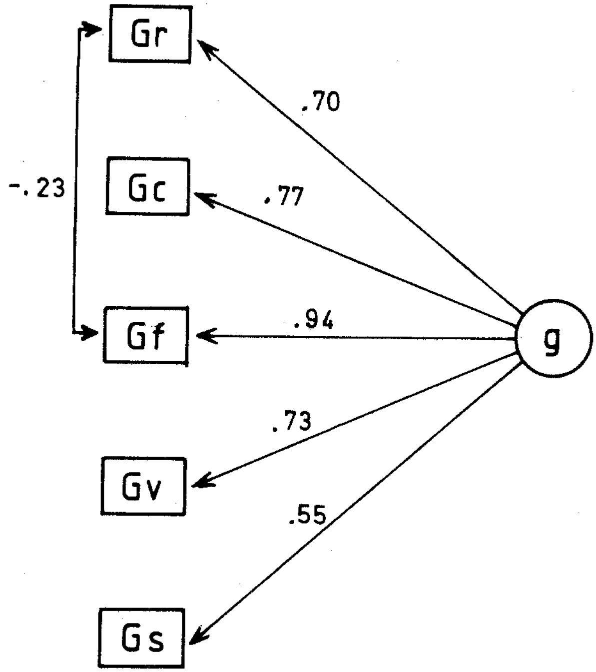

Figure 6. Final LISREL model for 15-year olds, Analysis B (Study III).

In this model, too, the loading of Gf in g is the highest one (.94) and the hypothesis that the relationship between Gf and g is unity could not be rejected, even though a border-line significance was observed (chi-square = 3.81, df = 1, $p < .051$).

Discussion

The LISREL analyses conducted according to approach A generally supported the hypothesized structure at the primary and secondary levels, even though there are rather large differences in the way in which the broad factors are represented in the models. This corresponds to the finding of exploratory factor analysis that broad factors technically may appear at different levels of factoring. The broad factors previously identified in simple structure factor analysis Gf, Gc, Gv, Gs, and Gr—were clearly recognizable in each of the three studies (Gr only in Study II and III) either as first-level latent variables or as second-level latent variables.

However, there are also some notable differences among the models. Among 11-year olds Gf relates closely to R and less so to CFR; the reverse is true in the 13 year-old group; while in the 15 year-old group, R is again central in the identification of Gf. A possible explanation of this is that in LISREL, as in exploratory factor analysis, the name-giving implies some structural similarity, not identity. Thus, since the test indices of R and CFR are somewhat different across the three studies, each primary factor may relate somewhat differently to a higher-order latent variable. Despite these variations, the Gf-factor found in each study remained in a central position relative to the other broad factors, and none of the studies caused rejection of the hypothesis of a perfect relationship between *g* and Gf.

As anticipated these analyses necessitated many modifications of initial models to reach acceptable fits, even including the deletion of variables previously central in the identification of some broad factors. While some of the relationships introduced in these modifications may prove replicable, we expect that most of them represent transient, and theoretically uninteresting influences, that will not replicate. However, even though it is difficult to see how these modifications could bias the models in favor of the hypothesis of equality between *g* and Gf, it is, of course, impossible to rule out the possibility that they may in one way or another have introduced bias into the models.

Minimizing the need of such post hoc "data fiddling" was the object of the alternative LISREL analyses according to approach B. The idea was that many, if not all, of these transient influences would be eliminated by the summing or marker variables for each primary factor. In these analyses, then, the LISREL modeling starts one or two steps higher in the hierarchy. In these analyses acceptable fits were obtained with only one or two modifications of the original model. And again, the hypothesis of one *g*-factor and a perfect relationship between *g* and Gf could not be rejected. The fact that the LISREL modeling in this approach comes fairly close to confirmatory analysis in the statistical meaning of the term should make this result more convincing.

It is quite interesting to note that the relationships among the higher-order factors seem quite robust to changes in the definitions of lower-order factors. Thus, in the models resulting from the application of approach A there is, as was noted above, a considerable variability in the definition of the first-order factors, as a function of differences in the sampling of tests and subjects. Still the higher-order factors seem reasonably invariant over the studies. This also holds true when the results obtained within approach B are brought into the picture, in spite of the fact that in this approach the lower-order factors are taken to be simple unweighted sums of observed variables.

It is, finally, interesting to relate our findings to the results of hierarchical order analyses using the Schmid-Leiman orthogonalizing procedure previously obtained on these three sets of data (Undheim, 1981b). In these simple structure factor analytic studies, only the performance of the 15-year olds (Study II above) showed the empirical equivalence of Gf and *g* (Undheim, 1981b). Undheim (1981b) attributed the lack of such equivalence in the other two sets of data to the sampling of variables, emphasizing in particular the need for a broadly based Gf-factor representative of the full Cattel-Horn conception of Fluid intelligence.

The present LISREL findings of such identity of Gf and *g* (even in the two cases of less than ideal variable sampling) may possibly be due to the fact that Gf was in fact somewhat more broadly defined in the LISREL analyses than in the original analyses. Thus, in Study I, Gf was in the LISREL analysis loaded not only by Figural Reasoning (CFR) and Memory Span (Ms), but also quite strongly by General reasoning (R). Also in Study II Gf relates more strongly to R, while in the original analysis there was an emphasis on the figural reasoning aspects of Gf. A more consistent use of summing indices for previously found primary factors might thus give Schmid-Leiman results more in line with the present LISREL findings.

In conclusion, then, the present studies are consistent with the findings of Undheim (1981b) and Gustafsson (1984) that Gf is equivalent with the *g*-factor of cognitive abilities as measured by factor-type intellectual tests. This finding carries several theoretical and practical implications, but discussions of these is beyond the scope of this paper (see however, Gustafsson 1980, 1982, 1984; Undheim 1981b, c, d, in press).

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