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The authors provide a brief review of the history and assessment of math anxiety, its relationship to personal and educational consequences, and its important impact on measures of performance. Overall, math anxiety causes an "affective drop," a decline in performance when math is performed under timed, high-stakes conditions, both in laboratory tests as well as in educational settings. This means that math achievement and proficiency scores for math-anxious individuals are underestimates of true ability. The primary cognitive impact of math anxiety is on working memory, particularly problematic given the important role working memory plays in math performance. The authors conclude with a discussion of risk factors for math anxiety and some factors to be kept in mind when working with math-anxious students.

Keywords: *math anxiety; math achievement; working memory; affect*

Mathematics anxiety is a person's negative affective reaction to situations involving numbers, math, and mathematics calculations, "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (Richardson & Suinn, 1972, p. 551). Math anxiety reactions can range from mild to severe, from seemingly minor frustration to overwhelming emotional (and physiological) disruption; for example, we had a college student burst into tears during a lab experiment that tested simple subtraction facts (Ashcraft, 2002). People can experience math anxiety in formal settings, in a math classroom or when taking a high-stakes standardized math test, or in more everyday settings, for example when trying to balance a checkbook or figure a tip on a restaurant bill when others are watching.

Research on math anxiety has traditionally examined its relationships with other affective constructs, such as test or generalized anxiety, with various academic outcomes such as school grades or with motivational factors such as intent to take additional math courses or pursue math-related college majors (e.g., Hembree, 1990). Our research on the topic has taken a different approach, investigating the possible cognitive consequences of math anxiety when people perform math-related tasks in the laboratory. Taken together, these lines of research suggest strongly that math anxiety is a significant impediment to math achievement, one that affects a considerable portion of the population and one that merits serious attention both in terms of assessment and intervention.

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The purpose of this article is to outline some of the important findings about math anxiety and to indicate how math anxiety relates to other important performance measures, both achievement tests and cognitive tests of math processing. We explain current thinking about the mental mechanism thought to underlie the affective drop in performance mentioned in the title, and then, we explore a model that suggests some of the factors that may place students at risk for math anxiety. We do not claim that math anxiety is a genuine case of mathematics learning disability, in any traditional sense of the term *disability* (e.g., Mazzocco, 2007). We do, however, suggest that math anxiety *functions* as a disability in the sense that there are well investigated—and negative—personal, educational, and cognitive consequences of math anxiety. Unfortunately, these negative consequences affect a substantial percentage of the population (e.g., Ashcraft, 2002; Ashcraft, Krause, & Hopko, 2007).

Historical Background and Assessments

Two reports in the 1950s were instrumental in triggering modern interest in the topic of math anxiety: first, an anecdotal report by a teacher who commented on her students' emotional difficulties with math (Gough, 1954) and second, a surprisingly insightful paper on "numerical anxiety" by Dreger and Aiken (1957). Dreger and Aiken added three math-related items to the Taylor Manifest Anxiety Scale and renamed it the Numerical Anxiety Scale, thus providing the first objective test of math anxiety. They also made three general predictions: (a) that math anxiety would be different from general anxiety, although the two would overlap to a degree; (b) that math anxiety would not be systematically related to general intelligence, although it would probably correlate negatively with quantitative scores on IQ tests; and (c) that there would be an inverse relationship between math anxiety and academic performance in math coursework. All three of these predictions have been confirmed repeatedly by subsequent research (see Hembree, 1990).

A major advance in the research occurred when Richardson and Suinn (1972) published a general purpose assessment instrument for math anxiety, the Mathematics Anxiety Rating Scale, commonly called the MARS. The MARS is a 98-item rating scale on which respondents rate, on a 1 to 5 Likert-type scale, how anxious they would feel in situations ranging from formal math settings (e.g., taking a pop quiz in math class) to informal, everyday situations (totaling a dinner bill you think has overcharged you). Given its objectivity, availability, and accompanying psychometric data (e.g., the 2-week test-retest reliability was .85), the MARS became the test of choice for those investigating math anxiety.

For research or assessment purposes, however, the 98-item MARS is burdensome. It is time consuming to administer, cumbersome to score (scores range from 98 to 490), and given that shorter instruments are now available, far too long. Several descendants have emerged more recently that maintain good reliability and reduce administration time. For example, in our laboratory work with college students, we have used a 25-item version of the MARS, which we call the sMARS (shortened MARS; e.g., Ashcraft & Kirk, 2001, based on Alexander & Martray, 1989). It is a convenient and quick way of assessing a participant's level of math anxiety, available at no cost in the Alexander and Martray (1989) publication, which correlates very strongly (.97) with the original MARS. Hopko, Mahadevan, Bare, and Hunt (2003) have recently constructed the nine-item Abbreviated

Math Anxiety Scale (AMAS) test. Two-week test-retest reliability was .85 for this test, and factor analysis yielded a two-factor structure, with factors of learning math anxiety and math evaluation/math test anxiety accounting for 70% of the overall variance in scores.

Both the sMARS and the AMAS can be used with high school students, although enough of the items refer to advanced topics (e.g., algebra, graduation requirements) that younger respondents probably could not respond meaningfully. In passing, we note that in our work with college students, we also ask a one-item question: “On a scale from 1 to 10, with 10 being the most anxious, how math anxious are you?” Our results from this question have correlated anywhere from .48 to .85 with sMARS scores. Although we do not advocate using this one question instead of a validated survey like the AMAS, it does suggest that a very quick, informal direct question can often be used as a preliminary screen for math anxiety, with some degree of validity.

Younger respondents can be given one of the age-appropriate descendants of the original MARS—for example, the MARS-A (for adolescents) or the MARS-E (for elementary school students, starting with fourth graders), although these tests are now rather dated. In our preliminary work with grade school children, we are exploring alternatives to our one-question statement, as very young children are likely unfamiliar with adult-like connotations for terms such as *anxious* (or possibly *math* as opposed to *arithmetic*). Based on a similar attitude scale about children’s reading, which used drawings of Garfield with facial expressions ranging from happy to sad, we simply provide children with a row of schematic faces, a classic “smiley” face down through a “frowney” face, and ask “How do you feel when you do arithmetic in school?” It is not yet clear whether this alternative elicits anxiety-related responding or responding based on some other factor.

A more formal definition of math anxiety is necessary for research, however, much as formal definitions of mathematics learning disability are needed for assessment and intervention (see Mazzocco, 2007). Using the sMARS as our example, respondents answer the 25 items by responding on a 5-level Likert-type scale from *no anxiety at all* up through *extreme anxiety*. We then award 0 through 4 points, respectively, for their responses and then sum the points, yielding a possible range of 0 to 100. Across many college samples, the overall grand mean has been 36.0, with a standard deviation of 16. For purposes of forming low, medium, and high math-anxious groups, we have simply established statistical cutoffs based on these values. For the low math anxiety group, we count scores at or below 1 *SD* below the grand mean, 20 or lower, and for the high-anxious group, scores at or above 1 *SD* above the mean, 52 or higher. For the medium-anxiety group, we center the 16-point *SD* on the grand mean, such that this group’s scores range from plus or minus 0.5 *SD* around the mean, from 28 to 42. Based on the normal curve, roughly 17% of the entire population would be expected to be low in math anxiety, 17% would be high in math anxiety, and 38% of the population should fall within 0.5 *SD* of the mean.

This is a purely statistical definition, however, used for purposes of forming groups in a research design. As such, it is clearly mistaken to draw conclusions such as “17% of the population is high in math anxiety.” Instead, it is important to use this statistical criterion as an initial guide and to then refine those definitions by examining actual performance data to see how performance depends on an individual’s actual level of math anxiety. We turn to that topic shortly.

Relationships to Other Factors

One possible reaction to a student's math anxiety—whether the student confesses to having math anxiety or the teacher "diagnoses" it—is to merely acknowledge it and then say something like "Well, you'll just have to deal with that" or "You should be able to overcome that," or even "So what?" These reactions fail to appreciate the pervasive relationships that can exist between math anxiety and other aspects of personal, educational, and cognitive functioning. A large literature now exists on how math anxiety relates to aspects of personality and educational attainment, and we review that literature briefly here. We then turn to our research work on some cognitive consequences of math anxiety, which are relevant to the actual doing of math *in the moment*—in the classroom during a test, in the lab during a cognitive test of math performance. Given society's increasing reliance on standardized testing, knowing how math anxiety affects performance as students actually perform math, or as they are assessed in math, is extremely important.

Hembree's (1990) and Ma's (1999) articles provide meta-analyses of the existing literature on math anxiety, with Ma's article focusing exclusively on precollege samples. The articles summarize what is known about the relationship between math anxiety and a variety of other personal and educational factors. To begin with, math anxiety correlates only –.17 with overall IQ and a nonsignificant –.06 when only the verbal aptitude sections of standardized tests are used (except as noted, all correlations reported in this section are from the Hembree and Ma meta-analyses). Clearly, math anxiety is not a correlate of intelligence. Math anxiety does correlate with several other anxiety measures, most strongly with test anxiety (.52) but moderately with general, trait, and state anxiety (respectively, .35, .38, and .42). Most researchers interpret this to mean that test anxiety is a strong correlate of math anxiety and that math anxiety is separate from, although related to, other anxiety constructs.

Correlations between math anxiety and several math attitude measures are uniformly negative and frequently quite strong—for example, with enjoyment of math in precollege samples (–.75), with self-confidence in math in precollege samples (–.82), and with motivation in math (–.64). The correlation between math anxiety and ratings of the usefulness of math is also negative (–.37), as is the correlation with ratings of math teachers (–.46). Negative conversational remarks about math are relatively common in our culture, of course, but persistent, negative remarks about oneself and math, especially those that display dislike and low self-confidence, are probably diagnostic.

Turning to measures that examine educational effects, math anxiety correlates –.34 with math achievement scores in precollege samples (and –.31 in college samples), –.30 with high school math grades, and –.27 with college math grades (Ashcraft & Kirk, 2001). And it correlates –.31 with the extent of high school math courses taken (electives) and –.32 with intent to enroll in more math courses (college; see Ashcraft et al., 2007, for a more complete discussion of these correlational results). We return in a moment to the negative correlation with math achievement, given its centrality to educational assessment.

In sum, the higher one's level of math anxiety, the lower one's score is on math achievement tests, the fewer math courses one takes, and the lower one's grades are in the math courses that are taken. Furthermore, with higher math anxiety, one's attitudes about math are poorer, including one's intent to take further math courses. This of course influences decisions in college concerning majors and career paths. Predictably, those majoring in physical sciences, engineering, and math in college have low math anxiety, and those in humanities and non-math-related disciplines have higher math anxiety. Particularly worrisome, the college major with the highest level of math anxiety is elementary education (Hembree, 1990).

A convenient way of summarizing much of this correlational research is to appeal to the concept of "avoidance," saying that avoidance of math is an overriding characteristic of math-anxious individuals. Math-anxious individuals avoid taking math courses whenever possible, avoid selecting courses of study in college that involve math, and of course avoid career paths that involve math. It seems likely, although this has not been tested yet, that such avoidance would be observable in the math classroom as well, in terms of student engagement in the lesson, participation in class, time spent on studying, submission of homework, and so on. Turner et al. (2002) reported that avoidance was the typical student behavioral pattern in response to teachers whose teaching style was cold and unsupportive; students avoided eye contact during class, avoided out-of-class make-up and help sessions, and the like when they had such teachers. Such avoidance seems only one step removed from math anxiety, given some of the risk factors to be discussed below.

Relationships to Performance

In the published literature, no other relationship is as troublesome as the negative correlation between math anxiety and math achievement; in college samples, the correlation is –.31 (and –.34 in precollege samples; Hembree, 1990). From the standpoint of doing research on math anxiety, the relationship means there is a confound between the two—any time a high math-anxious group performs more poorly than a low-anxious group, the interpretive question is whether their poor performance was due to high anxiety or to their lower level of mastery of the math. Our solution to this vexing problem in the lab has been to test college students on fairly simple arithmetic (e.g., two-column addition problems) on which the anxiety groups have been shown not to have differences in mastery when they are tested in an untimed, low-pressure setting (Faust, Ashcraft, & Fleck, 1996; see also Ashcraft et al., 2007, who showed no math anxiety differences on accuracy using the first three lines of the Wide Range Achievement Test, the three lines that test whole number arithmetic).

But the relationship between math anxiety and math achievement is more knotty than this. We typically measure not only a student's aptitude and achievement with standardized tests but also the effectiveness of instruction and indeed the quality of education provided by the school. As such, it is important to know how math anxiety affects performance, how it alters how people actually perform math in the moment of doing it. This is the line of research we have pursued in our lab for several years, examining how cognitive processing is affected by math anxiety. To foreshadow our results, we find evidence for what might be termed an *affective drop*, a drop in performance that can be attributed to math anxiety independent of the individual's competence or achievement in math.

In our early research (Ashcraft & Faust, 1994; Faust et al., 1996), we found that math anxiety had very little effect on performance to the simplest kinds of arithmetic: the basic facts of addition and multiplication; for the most part, college students perform such problems in a fairly automatic fashion, by means of memory retrieval. More complex arithmetic problems, however, showed either slower performance or considerably less accurate performance for the higher anxious participants. In one condition, which tested difficult problems in all four arithmetic operations, our highest anxiety group performed as rapidly as the low-anxiety group but made twice as many errors. This tends to be the typical pattern; participants of medium and high math anxiety respond quite slowly, compared to low-anxious participants, and/or perform quite inaccurately whenever we ask them to perform beyond the level of single-digit arithmetic. This is the affective drop in performance that we observe in the lab, the kind of drop that will detract from performance on typically administered tests of math proficiency and achievement. Importantly, we find this drop for both medium- and high-anxious groups, composing somewhat more than half of our participants.

In the most theoretically meaningful result we have obtained (Ashcraft & Kirk, 2001), we tested college adults on two-column additions, showing them problems with and without carrying and having them furnish the answers out loud in a timed setting. On a portion of the trials, they simultaneously had to hold either two or six unrelated letters of the alphabet in working memory while doing the addition problem and then report the letters back in order. This is referred to as the "dual task" method in cognitive psychology, taxing working memory by performing two tasks at once.

In brief, our results showed that two-column addition, especially when it involved carrying, placed a heavy load on working memory, as shown by higher errors in the letter recall task. Most important, the increase in letter recall errors was especially pronounced among the high math-anxious participants. Our interpretation was relatively straightforward. High math-anxious individuals are already using some of their limited working memory resources worrying about their anxiety whenever they perform a math task. When given an especially taxing math task, say one involving carrying, the load on working memory becomes even more intense. Coupling this with the letter recall task led to serious difficulties. It was as if the high math-anxious participants were participating in a three-way competition for their limited working memory resources: difficult math, letter retention and recall, and their own math anxiety. The load on working memory became so pronounced that their performance deteriorated markedly—affective drop.

Extrapolate this situation to high-stakes testing, say a standardized math test needed for college entrance or, quite common these days, a math proficiency test needed to obtain a high school diploma. The pressure to perform well is intense, the math itself is challenging, and the student must still grapple with the internal worries and fears associated with the math anxiety. Just as found in the lab, the student's math anxiety compromises the essential working memory resources necessary for successful completion of the math problems. In testing situations, we typically interpret a student's score on the standardized test as an indicator of the student's mastery of math, literally of his or her math achievement (or of the quality of instruction). Based on our research findings, this is an interpretive mistake. In the pressure-inducing, high-stakes setting, some portion of the math-anxious student's low score is more appropriately attributable to math anxiety, to the same kind of affective drop we see in the lab.

Consistent with this interpretation, Hembree (1990) reported a fascinating finding in his meta-analysis. High math-anxious individuals who had undergone cognitive behavioral interventions for their math anxiety subsequently showed math achievement scores "in the normal range." On the assumption that the therapeutic intervention did not involve teaching any math, this clearly suggests that their pretherapeutic math achievement scores were underestimates of their true math achievement. Reducing math anxiety thus removed an impediment to adequate performance on the achievement tests. The message for assessment seems very clear here: Math anxiety reduces a person's assessed math achievement regardless of the person's true level of mastery of math. This is the same kind of affective drop that our dual task study of college students showed in two-column addition.

We wish to repeat an important point here regarding the cognitive component that seems especially vulnerable to the effects of math anxiety. It was working memory that was compromised in our study of college students' performance and math anxiety; working memory suffered the brunt of the math anxiety effect because of the inner-worries and self-doubts that are reported by math-anxious individuals. This is the same cognitive mechanism at work when females' math scores drop because they have been exposed to a negative stereotype concerning women's allegedly poorer performance in math—the stereotype threat effect (e.g., Beilock, Rydell, & McConnell, 2007; see also Eysenck & Calvo, 1992). The involvement of working memory is important to note because math becomes especially abstract and difficult when it involves multistep computations, sequencing of mental procedures, mental look up of formulas and equations, and the like. All of these factors place a heavy load on working memory, as documented in the literature (e.g., LeFevre, DeStefano, Coleman, & Shanahan, 2005).

A Model of Math Anxiety Susceptibility—Risk Factors

Given the wealth of information about correlates of math anxiety, it is somewhat surprising that little if any research has been reported concerning its onset or possible causes. Much is known developmentally, of course. For example, across Grades 6 through college, females score approximately 0.3 *SD* higher on math anxiety scales (Hembree, 1990; Hopko, 2003) than males, although it is still not clear that they display greater disruption in math performance as a consequence of this. Math anxiety peaks at the Grade 9 to 10 level and then essentially flattens thereafter (see Figure 1, Hembree, 1990). Hardly any research has examined children younger than the fourth grade (Hembree, 1990; Ma, 1999). But there appears to be no empirical information about the onset of math anxiety or firm evidence about why it grows through the middle school years.

Partly as an attempt to prompt research on this topic, we have recently proposed a model of math anxiety susceptibility and highlighted several plausible risk factors (Ashcraft et al., 2007). Our model predicts that a child with adequate skill in math, adequate motivation, and adequate working memory will exhibit adequate mastery and performance in math and in particular will not fall prey to a set of cognitive biases that disrupt responding. Risk factors for math anxiety, on the other hand, would be low skill or ability in math, inadequate motivation, or insufficient working memory.

Specifically, a child with lower than normal math skill will struggle with math concepts, possibly in the early math curriculum but certainly once the curriculum advances beyond simple arithmetic. Such a child may show early deficits in "number sense," a warning sign given the growing body of evidence that shows how critical number sense is in early schooling and how predictive it is of math achievement (e.g., Jordan, Kaplan, Olah, & Locuniak, 2006; Siegler & Booth, 2004). More generally, we predict that children with lower than average math ability will fall behind their peers, receive negative feedback from teachers and parents, and be more likely to develop negative attitudes and motivations about math. Likewise, a child with lower than average working memory capacity is one who will struggle with more complex arithmetic and math, given the importance of working memory to performance in math and other educationally relevant cognitive skills (Engle, 2002; LeFevre et al., 2005).

In the model, these risk factors by themselves can lead to performance deficits and avoidance; for instance, low math ability and consequent poor motivation could easily lead a student to avoid elective math in high school, a pattern that would also detract from performance on standardized tests that cover advanced topics. But the model also suggests that several cognitive biases may also result, leading to math anxiety that co-occurs with the performance deficits and avoidance. In particular, there may be an increase in negative math attitudes and self-focused attention in some children, internalizing the negative feedback received from poor performance. In some children, there may also be a heightened vulnerability to anxiety, or vulnerability to public embarrassment, that becomes associated with math; anecdotally, we have had several college students tell us that their anxiety stemmed from having to work a difficult math problem on the blackboard during math class and the resultant embarrassment they felt in front of their peers and teachers. The "cold, unsupportive" teacher style mentioned above (Turner et al., 2002) may feed into this vulnerability too, although there is no way of knowing how common such a style is in math classrooms.

In Practice

We conclude briefly with three points aimed at practice and assessment. First, our preliminary data suggest that children in the first 2 or 3 years of elementary school are not troubled by math anxiety (or do not report it), although by fourth and fifth grade some start to indicate a degree of apprehension. Thereafter, and certainly whenever there are other indicators of math anxiety, it is not possible to interpret a student's score on a standardized math test as a clear indicator of math ability or achievement. Our evidence indicates that every time a math-anxious individual is asked to perform math in a timed, high-stakes setting, the individual's math anxiety is aroused and causes *affective drop*, a significant decline in performance. Indeed, Hopko's (2003) interpretation of factor analysis results suggests that *any* math test arouses anxiety. The score on the test is simply not an accurate reflection of the respondent's ability or achievement.

Second, it is likely that a student's math anxiety is aroused in the math classroom itself, possibly only to a minor degree during routine class activities but almost certainly when the student is called on to answer a question or solve a problem. It is certainly aroused when the student takes a test. Thus, class grades and in-class assessments are also suspect as indicators of true achievement, at least to the extent that they rely on timed tests completed under stress or pressure. Overall, it seems more likely than not that the math-anxious student learns somewhat less in the math classroom than the nonanxious student, although testing for this differential learning is obviously difficult because of the anxiety drop factor discussed here.

Finally, the existing literature is quite clear on the long-term effects of math anxiety. Math-anxious individuals avoid elective math coursework, avoid college majors that require math, and avoid career paths that involve math. Given society's increasing reliance on technology and current concerns over STEM (science, technology, engineering, and math) training, anything that can be done to investigate, understand, and thereby prevent math anxiety is to be encouraged, beginning with promoting a sensitivity to its effects on students in the K-12 math classroom.

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